

Reimbursing bus operators

Analysis of the DfT's guidance

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1 Introduction

The Concessionary Bus Travel Act 2007 extended the statutory minimum entitlement for free off-peak local bus travel for residents over 60 and disabled passengers to local bus travel anywhere in England from April 1st 2008. In the context of this extended bus travel entitlement, the Department for Transport (DfT) issued guidance to local authorities in England on reimbursing bus operators for providing this service ('Concessionary Travel for Older and Disabled People: Guidance on Reimbursing Bus Operators').¹ This guidance is accompanied by a 'reimbursement analysis tool' (RAT), a spreadsheet tool to quantify the reimbursement due to bus operators.

According to the DfT guidance, bus operators are reimbursed for the forgone revenue (ie, the revenue the operator would have received in the absence of the scheme) and for the additional costs generated by the extended bus travel entitlement. In calculating the forgone revenue and the additional costs generated by the extended entitlement, the DfT makes a number of assumptions about the shape and the drivers of the concessionary travel demand; the price elasticity of demand, including the evolution of the elasticity over time and its relation with changes in the fare; the behaviour of existing and new pass-holders; and the additional costs generated by the scheme.

This report presents an analysis of the DfT guidance and the RAT. In analysing the guidance, the rationality of the overall reimbursement scheme is first considered, alongside the evidence presented by the DfT to support its assumptions. To assess these assumptions, Oxera has reviewed the economic literature quoted by the DfT, and conducted simulated exercises to determine the directional impact that a change in the assumptions could have in the reimbursement due to bus operators.

The following conclusions can be drawn about the guidance.

- The DfT methodology is based on assumptions about the nature of demand for bus travel. These assumptions are fairly arbitrary, cannot be justified unambiguously by empirical evidence, and involve extrapolating experience with current levels of prices to zero prices in a way that is highly dependent on the precise method used.
- While there is some evidence regarding the use of a negative exponential function, other functional forms with variable elasticity of demand could also be used (eg, a linear demand). If other functional forms are used, the DfT's proposed revenue reimbursement formula does not hold for estimating the number of non-generated trips.
- The evidence presented by the DfT to support its assumption that the impact of a 1% increase in real fares is 0.2% is limited. This assumption has important implications for the bus operators, as values lower than the assumed 0.2% could have a large impact on the reimbursement factor. There is also a potential inconsistency in the application of this adjustment as it could mean that the DfT is using two different demand curves: one for calculating the elasticity and one for calculating the effects of the scheme.
- The evidence presented by the DfT to assume that the long-run elasticity doubles the short-run elasticity is limited. Related to this point, little evidence is presented to justify that the effect of the scheme follows a specific speed of adjustment over a five-year period.

¹ DfT (2007), 'Concessionary Travel for Older and Disabled People: Guidance on Reimbursing Bus Operators', December.

- On the cost side, no support is provided to sustain the assumption that the range of additional costs is between £0.01 and £0.15 per trip generated.

This report is structured as follows.

- Section 1 summarises the DfT guidance and the working of the RAT and highlights the main assumptions used by the DfT.
- Section 2 reviews the assumptions regarding the shape of the demand function and their implications for the operators' revenue reimbursement.
- Section 3 reviews the assumptions regarding the elasticity.
- Section 4 discusses other aspects of the RAT that could affect the revenue reimbursement.
- Section 5 sets out the conclusions.

2 DfT guidance and the RAT: a description

The principle underlying the DfT guidance is that operators should be left 'neither better nor worse off' as a result of the existence of the concessionary travel scheme. This means that operators should be compensated for the revenue forgone as a result of the scheme, as well as for the additional costs imposed by it. The following sections describe how these two elements are calculated, and highlight the main assumptions used by the DfT.

2.1 Calculation of the forgone revenue

To estimate the forgone revenue, it is necessary to identify the number of trips that would have occurred in the absence of the scheme (ie, the 'non-generated trips'). These non-generated trips are not directly observable for bus operators, and therefore need to be inferred from the total numbers of trips observed after introduction of the scheme. This is done by adjusting the total number of trips under the free-fare concession by a 'reimbursement factor' (RF):

$$\text{Non-generated trips} = (\text{total number of trips}) \times (\text{RF}) \quad \text{Equation 2.1}$$

Having determined the number of non-generated trips, the forgone revenue is calculated as:

$$\text{Forgone revenue} = (\text{non-generated trips}) \times (\text{average fare}) \quad \text{Equation 2.2}$$

Where the average commercial fare is the nominal fare that would be applied in the absence of the scheme, based on a basket of tickets.

A critical assumption behind the calculation of non-generated trips is that the demand for bus travel takes the form of a negative exponential function. This demand function has three properties:

- the elasticity is proportional to the price (ie, the lower the fare, the lower the elasticity in the absolute value);
- the quantity of trips is finite at a zero fare; and
- the RF is equal to $e^{\text{elasticity}}$ and therefore the RF increases as the fare (elasticity) decreases.

Although there is some evidence to support the use of a negatively exponential demand function instead of a demand function with a constant elasticity, there is less evidence to support the selection of a negative exponential demand function in favour of other functional forms with variable elasticity (eg, a linear demand function).² As explained in section 3 below, the DfT's proposed formula for the calculation of non-generated trips does not hold under alternative assumptions regarding the shape of the demand.

A second assumption relates to the calculation of the elasticity in Equation 2.1. In a negative exponential demand function, the elasticity is proportional to the price. However, the DfT has assumed that, in practice, this relationship is less sensitive than the direct relationship suggested by the negative exponential demand curve. In particular, the DfT has assumed that a 1% real fare rise could be expected to increase the fare elasticity by 0.2%.

² Dargay, J. and Hanly, M. (2002), 'The demand for local bus services in England', *Journal of Transport Economics and Policy*, 36, January.

The DfT also assumes that the number of non-generated trips reduces gradually up to the fifth year of the scheme. It assumes that 50% of the generated trips will occur in the first year, 75% within two years, 88% within three years, 94% within four years and broadly 100% within five years. This assumption is incorporated into the RAT by increasing the elasticity on an annual basis. In the RAT, the default increases in elasticity are 0% in the first year, 25% in the second year, 38% in the third year, 44% in the fourth year and 50% in the fifth year. The effects of these assumptions on the bus operators' revenue reimbursements are considered in section 4.

2.2 Calculation of the additional costs

An assessment of the basis for the cost allowance is outside the scope of this analysis, but the DfT-recommended approach is set out below for completeness. According to the DfT guideline, three cost categories are considered for reimbursement:

- marginal operating costs;
- capital costs;
- other costs (eg, the scheme implementation costs).

The principle established in the DfT guidance for estimating additional costs is that operators should present evidence to support these costs. However, in the absence of evidence, the DfT recommends using a range of additional costs of between £0.01 and £0.15 per generated trip, including an element of capital costs. No evidence is presented in the DfT guidance to support the suggested range of additional costs.

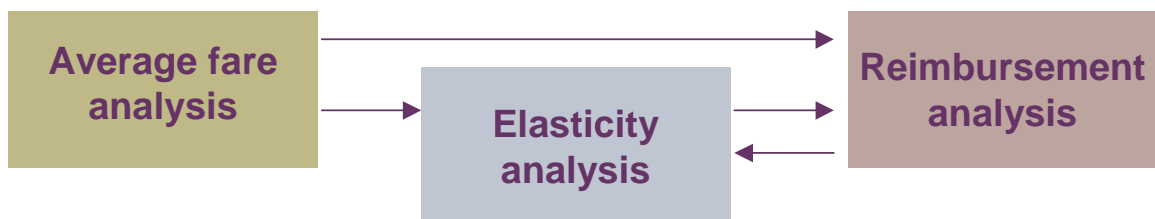
In the RAT, the additional costs of the generated trips are estimated as follows:

$$\text{Cost} = (\text{generated trips}) * (\text{cost per generated trip}) + (\text{other costs}) \quad \text{Equation 2.3}$$

2.3 Reimbursement analysis tool

The RAT enables the reimbursement due to bus operators to be quantified, and comprises three linked modules: average fare analysis, elasticity analysis and reimbursement analysis (see Figure 2.1).

Figure 2.1 Overview of the RAT



Source: Oxera analysis based on RAT.

The final output of the RAT is calculated in the reimbursement analysis module, using inputs from the average fare and elasticity analysis components. The calculations in each module require the bus operator to provide several inputs. The workings of the RAT are described below.

2.3.1 Average fare analysis

Price elasticity in the RAT is calculated for the average fare of a basket of tickets instead of quoted fares. The average fare analysis spreadsheet calculates the average fare. This is used twice in the process: first to calculate the average commercial fare in Year 1 (before the

change from half/flat fares to zero fares) and second, to calculate the average (expected) fare that the bus operator would have applied in the absence of the scheme (ie, from April 2008 onwards). The default template available in the average fare analysis module considers several ticket types that vary with the trip distance (eg, single tickets for less than one mile, single tickets for more than one mile, return tickets for less than one mile, return tickets for more than one mile, daily pass, and weekly pass). In this context, bus operators with a high proportion of trips corresponding to long-distance ticket types may have a higher average fare than operators with a high proportion of trips corresponding to short-distance tickets.

2.3.2 Elasticity analysis

This part of the RAT estimates the average full fare elasticity based on historical data. The most important part of this calculation consists in isolating the effect of changes in fares from other variables, such as the long-term trend in bus patronage, change in substitutes, change in quality of service, and differences in the trip rate between new and existing pass-holders. The remaining changes in demand are attributed to the fare reduction and are therefore used to calculate the elasticity.

For this calculation, bus operators are required to input their views on the following parameters.

- Impact on the number of trips (as a percentage of Year 1 trips, corresponding to the number of half-/flat-fare trips) from underlying bus travel trend. This is an allowance to reflect the long-term trend in concessionary bus patronage.
- Impact (as a percentage of Year 1 trips) from changes to substitutes. This accounts for changes in trips due to changes in substitutes to concessionary bus travel. For example, if car travel becomes less attractive, this will result in an increase in bus travel trips, regardless of the change in fares.
- Impact (as a percentage of Year 1 trips) from changes to bus quality of service. This reflects changes in trip numbers due to changes in bus quality (eg, more low-floor buses), and supply (eg, more buses on a particular route). A positive percentage indicates that changes to bus quality or supply resulted in an increase in the number of trips between Year 1 and Year 2.
- New pass-holder trip rate in Year 2 expressed in terms of trips per pass-holder. The DfT does not specify a value for this parameter, although it suggests the direction that this parameter should take. In particular, it recommends using the trip rate of existing pass-holders as a starting point, but expects that the trip rate of new pass-holders will be broadly around half that of existing pass-holders, although with a large range, where much lower rates could still be plausible.

In estimating the elasticity, information about the changes from half/flat fare in 2006/07 to free fare could be used. In this case, the adjustment for bus travel trend, change in substitutes and change in quality are applied to the number of half-fare trips. For example, if there is evidence of a negative trend in concessionary bus patronage (ie, that, even with no change in fare, the number of trips would have decreased), then it is necessary to add the change in number of trips due to this trend back to the estimate of the change in trips due to the fare change.

2.3.3 Reimbursement analysis

This module of the RAT calculates the reimbursement due to an operator for a given period under various assumptions: using the elasticity derived from the elasticity analysis module and key DfT assumptions; using the elasticity derived from the elasticity analysis module but varying key assumptions on the basis of local evidence; and inputting a Transport

Concession Authority-calculated elasticity (ie, not using the elasticity analysis module to calculate the elasticity).

In this module, the bus operator is required to input the following information for the year for which the reimbursement is calculated:

- average commercial fare for the year for which the reimbursement is calculated (ie, 2008 onwards);
- forecast or actual recorded concessionary trips for that year;
- an estimate of the additional costs (£) per generated trip and other additional costs not linked to the volume of generated trips.

The key parameters calculated in the reimbursement analysis module are the following.

- **The updated elasticity**—the elasticity calculated from the elasticity analysis module adjusted to account for the impact of real fare changes and the long-term trend in elasticity.
- **Reimbursement factor**—estimated as $\exp^{\text{elasticity}}$, where elasticity refers to the updated version of the elasticity.
- **Number of ‘non-generated trips’**—estimated as in Equation 2.1 above.
- **Revenue reimbursement**—estimated as in Equation 2.2 above.
- **Number of ‘generated trips’**—calculated as the difference between the total number of trips observed after the scheme and the non-generated trips.
- **Total additional cost**—the cost of providing the generated trips, calculated as in Equation 2.3 above.

The actual calculation of the reimbursement works intuitively in the following framework. It extrapolates the elasticity at full fare calculated in the elasticity analysis module, taking into account changes in real fares and the long-run effects of the scheme. This extrapolation of the elasticity between the years before and after the scheme depends on the assumption that the same elasticity applies to both curves (ie, that the elasticity at each fare level is the same for both groups).³

³ See DfT (2007), op. cit., p. 47.

3 Shape of the demand function

Any functional form that relates demand inversely to the price paid could in theory be used to characterise the demand. By way of introduction, it is possible to categorise functional forms into two types: variable elasticity and constant elasticity models. In variable elasticity models, the elasticity typically increases as the fare rises, and decreases as the fare falls. Variable elasticity models could include many different functional forms, such as a negative exponential, a linear or a negative cubic function.⁴

The DfT guidance recommends a negative exponential demand function. This choice of demand is based on Dargay and Hanly (1999) and the desirable properties of this function—in particular, that it is possible to define the RF as $\exp^{\text{elasticity}}$.⁵ However, the review of the academic literature on bus transport demand appears inconclusive as to whether the form of demand for bus travel is better represented by a negative exponential function than other possible formulations. This point is crucial because the DfT's proposed RF will not be valid with other demand functions. The literature reviewed is summarised below, together with an example showing that the DfT's proposed RF does not hold for other functional forms.

3.1 Review of literature on the functional form of the demand for bus travel

In selecting its preferred functional form, the DfT quotes Dargay and Hanly (1999)⁶ to indicate that the negative exponential demand function 'has performed rather better than other possible formulations'.⁷ However, Dargay and Hanly (1999) analysis is not as conclusive regarding the functional form of the demand for concessionary travel as the DfT appears to suggest.

First, the Dargay and Hanly (1999) study is concerned only with non-concessionary travel. As the authors indicate:

the study is concerned only with the non concessionary bus market. It is not the intention to investigate the effects of concessionary fares or fare changes, or to contribute to policy discussions relating to these.⁸

Second, Dargay and Hanly (1999) show that a constant elasticity demand function is a robust representation of the demand for bus travel in certain datasets (see Table 3.1 below). More importantly, in the case where a negative exponential demand function is recommended (ie, variable elasticity demand), this conclusion is based exclusively on the comparison of the statistical significance of the negative exponential demand function (semi-log model) and the constant elasticity demand (log linear model). No hypothesis testing has been performed between different variable elasticity demand models. Therefore, Dargay and Hanly (1999) do not necessarily indicate that the negative exponential specification is better than another variable elasticity specification, such as a linear demand function.

⁴ Negative exponential functional forms of the generic class of $q = a \cdot e^{-b \cdot p}$

Negative cubic demand functions could be of the form $q = -b(p - a)^3$

Linear demand functions would be of the form $q = a - bp$

Where a and b are constants, q is the level of demand and p is the fare in all three examples.

⁵ Dargay, J. and Hanly, M. (1999), 'Bus fare elasticities', November, Report to the DETR, ref. 99/26, ESRC Transport Studies Unit, November.

⁶ Dargay and Hanly (1999), op. cit.

⁷ DfT (2007), op. cit., p. 18.

⁸ Dargay and Hanly (1999), op. cit., Executive Summary, p. ii.

Table 3.1 Summary of Dargay and Hanly (1999)

Dataset	Conclusion regarding the demand function form
Estimation results at the national level—Great Britain as a whole	The constant elasticity model was preferred to the alternative specifications, which allow the elasticity to vary on the basis of statistical tests. Statistical tests were also performed to investigate the stability of the estimated elasticities over time, and in all cases, stability could not be rejected. There is thus no evidence that the elasticities vary either over time or with the level fare or bus patronage
Estimation results for GB regional data	In the case of the regional data, there does seem to be a significant relationship between fares and elasticity. The semi-log model is preferred statistically to the double-log (constant elasticity)
Estimation results for individual English Metropolitan areas	The constant elasticity specification is preferred to the alternative specifications on the basis of statistical tests. There is thus no evidence that the elasticities vary either over time or with the level of fare
Estimation results at the English county level	There is a statistically significant relationship between the elasticity and the fare level. Demand appears to be relatively insensitive to fare changes at low fare levels, but as the fare increases, demand becomes more sensitive to fares

Source: Dargay and Hanly (1999), op. cit.

In a more recent exercise, Oxera (2003) estimated both constant and variable elasticity models, and, on the basis of the analysis and prior assumptions, concluded that the negative exponential demand (semi-log model) was preferred to the constant elasticity model. However, as in Dargay and Hanly (1999), this conclusion does not imply that the negative exponential demand function is better than other variable elasticity specifications.⁹

Other relevant literature, such as the Black Book, indicates that there is no consensus on the functional form that should be used for a demand equation; instead, it recommends that this should be derived from empirical evidence. The Black Book does not recommend a specific functional form, but notes that many of the functions estimated are linear or log-linear.¹⁰ A linear model would give very different results from the DfT approach, with potentially higher compensation to operators, as is shown below.

In summary, in contrast to the DfT suggestion, a review of the literature is not conclusive regarding the use of the negative exponential demand function as opposed to other functional forms (eg, a linear demand function or even a constant elasticity function). As is shown in the next section, changing the assumptions on the shape of the demand function has important implications for the calculation of the revenue reimbursement.

3.1.1 Revenue reimbursement under other demand forms

This section uses a numerical example to provide a simple illustration of how different assumptions about the functional form could lead to different answers for the revenue forgone from the same observed trips at zero fare and the same assumptions about the average full fare.

Table 3.2 shows a hypothetical example where a linear equation may lead to a revenue reimbursement that is different from that suggested by a negative exponential function. The example crucially depends on the assumptions for the values of parameters a and b in each

⁹ Oxera (2003), 'The demand for public transport: British bus market elasticities', Report to 'The Demand for Public Transport' team, August.

¹⁰ Balcombe, R., Mackett, R., Paulley, N., Preston, J., Shires, J., Titheridge, H., Wardman, M. and White, P. (2004), 'The Demand for Public Transport: A Practical Guide', TRL Report TRL593.

functional form. As such, the example is intended to show the sensitivity of reimbursement calculations to the assumption on functional form rather than to suggest an alternative functional form or to suggest parameters in either of these functional forms.

Table 3.2 Illustration of the impact of changing demand shape assumptions

	Negative exponential	Linear
Functional form	$q = a \cdot e^{-b \cdot p}$	$q = a - bp$
General point elasticity	$-b \cdot p$	$-b \cdot \frac{p}{q}$
Value of a ¹	1	1
Value of b	1	0.2
Observed demand at price p ₂ =0	1.00	1.00
Derived demand at price p ₁ =1	0.37	0.80
Point elasticity at full fare (p ₁)	-1.00	-0.25
Revenue forgone	0.37	0.80

Note: The definitions of the parameters and variables set out in the top two rows of the table are as follows: a is a constant that affects the position of the demand curve. It could theoretically differ between the two functional forms, but here it is assumed to be equal, as this ensures that the intercept with the x axis (ie, the observed number of trips under the free concessionary fare) is equivalent in each column. b is also a constant that affects the slope of the demand curve. In the negative exponential form, this parameter represents the extent to which the elasticity varies with changes in the real fare. e is the mathematical constant, q is the quantity demanded by concessionary travellers, and p is the fare for concessionary travellers.

Source: Oxera.

4 Elasticity assumptions

Having chosen its preferred functional form, the DfT makes additional assumptions about the impact that changes in real fares has on the elasticity, the differences between the elasticity in the short and long run, and the rate at which the short-run elasticity adjusts to the long-run elasticity level.

The impact of these assumptions on the revenue reimbursement of bus operators is examined below.

4.1 Impact on the elasticity from changes in real fares

The DfT's guidance suggests that historical evidence indicates that the elasticity is less sensitive to changes in real fares than the direct negative exponential form would imply. A negative exponential demand relationship suggests that a movement along the demand curve as the result of a real fare increase would directly alter the elasticity by the same proportion as the fare increase. As the guidance considers that historical evidence does not show this direct relationship, it recommends making an adjustment by implementing a proportional elasticity constant, with a default value of 0.2.¹¹

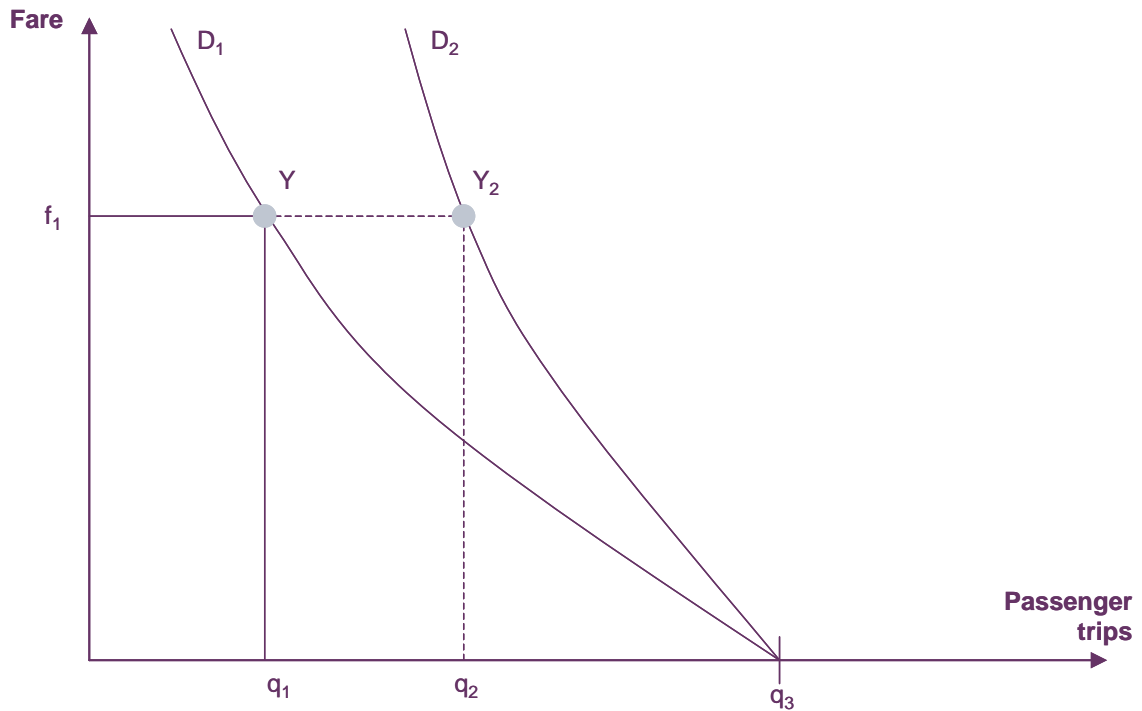
Effectively, this means that the relationship between price and demand is being written as:

$$q = e^{-bf} \quad \text{Equation 4.1}$$

where b is a constant that has a default value $b = 0.2$ and f is the fare. Figure 4.1 below shows the effect of using different values of b . The inclusion of a lower-value b makes the demand curve steeper. As such, the elasticity (of the new curve D2) becomes less sensitive to price changes at every price level, compared with the direct relationship demand curve (D1).

¹¹ DfT (2007), op. cit., paragraph 3.4.23.

Figure 4.1 Effects of changing the assumptions regarding the relationship between fare and elasticity



Source: Oxera.

In the 'elasticity analysis' module of the RAT, the elasticity at full fare of a change in fares from half/flat fare to zero fare is defined as follows:¹²

$$\text{Elasticity at full fare (n)} = -\ln\left(\frac{q_3}{q_2}\right) \times \frac{f_1}{f_2} \quad \text{Equation 4.2}$$

where f_1 represents full fare, f_2 half fare, q_3 the quantity of trips associated with a zero fare, and q_2 the quantity of trips associated with a half fare.

If the demand relationship is defined as in Equation 4.1 or equivalently $\ln(q) = -b \cdot f$, then it is possible to work out that the expression:

$$-\frac{\ln\left(\frac{q_3}{q_2}\right)}{f_2}$$

in Equation 4.2 is equal to the parameter $-b$.¹³ This means that the parameter b is determined by the values of q_3 , q_2 and f_2 input by the bus operators in the 'elasticity analysis' module of the RAT.

Regardless of the resulting value of parameter b in the 'elasticity analysis' module, in the 'reimbursement analysis' module of the RAT, the DfT sets the default value of parameter b for the purposes of measuring the impact of a change in real fares on elasticity as 0.2. This potentially creates an inconsistency, as it appears that two different demand curves are

¹² DfT (2007), op. cit, p. 47.

¹³ See Appendix A1 for a demonstration.

being used: one for calculating the elasticity (in the ‘elasticity analysis’ module) and one for calculating the effects of the scheme (in the ‘reimbursement analysis’ module).

In addition, setting the value for $b = 0.2$ has important implications for the revenue reimbursement of bus operators, as is shown in the next section. In particular, lower b values imply higher revenue reimbursement for bus operators. The DfT guidance provides little evidence for selecting a b value of 0.2. Other values of b could be used. In fact, the DfT guidance recognises this by allowing Transport Concession Authorities to input different values for b in the RAT.

4.2 Revenue reimbursement under different assumptions regarding the relation between elasticity and real fares changes

Table 4.1 provides a numerical example to examine the impact of changing the proportional elasticity parameter (denoted by the letter b). The proportional elasticity parameter represents the extent to which the elasticity varies with the changes in the real fare. Conceptually introducing a value such as this alters the slope of the demand curve. The example in Table 4.1 examines what happens when b is changed from 0.2 (the DfT selected value) to 0.1. As expected, lowering the proportional elasticity parameter increases the reimbursement due to operators.

Table 4.1 Illustration of the impact of changing the assumptions regarding the relation between elasticity and real fare changes

	Negative exponential ($b = 0.2$)	Negative exponential ($b < 0.2$)
Functional form	$q = a \cdot e^{-b \cdot p}$	$q = a \cdot e^{-b \cdot p}$
General point elasticity	$-b \cdot p$	$-b \cdot p$
Value of a	1	1
Value of b	0.2	0.1
Demand at price $p_2 = 0$	1.00	1.00
Demand at price $p_1 = 1$	0.82	0.90
Point elasticity at full fare	-0.20	-0.10
Forgone revenue	0.82	0.90

Note: The definitions of the parameters and variables set out in the top two rows of the table are as follows: a is a constant that affects the position of the demand curve. It could theoretically differ between the two functional forms, but here it is assumed to be equal, as this ensures that the intercept with the x axis (ie, the observed number of trips under the free concessionary fare) is equivalent in each column. b is also a constant that affects the slope of the demand curve. In the negative exponential form here, b represents the extent to which the elasticity varies with changes in the fare. e is the mathematical constant, q is the quantity demanded by concessionary travellers, and p is the fare for concessionary travellers.

Source: Oxera.

4.3 Long-run versus short-run elasticities

Differences between the long- and short-run elasticities could affect revenue reimbursement in either direction, depending on how the change is implemented. For example, a revision that led to the short-run elasticity being implemented over the whole period would increase the revenue reimbursement, while implementing the long-run elasticity over the whole period would reduce the reimbursement.

There is a degree of dispute over the difference between short- and long-run elasticities in the case of concessionary travel. For example, Goodwin (2006)¹⁴ concurs with the tone of the MVA (2003) report, which expressed some doubt as to whether long-run elasticities really were significantly higher.¹⁵ In addition, the elasticity values quoted by the DfT of 0.4 in the short run and 1.0 in the long run are based on the Black Book's meta-study. The Black Book notes that its long-run estimate is higher than some other studies, and that this may be because of the pre-dominance of cross-sectional studies in its meta-study, which tend to give higher values than those derived from time-series estimation.

Although DfT acknowledge that the difference between the long- and short-run elasticity in concessionary travel might be low compared with other types of travel, its final assumption is that the long-run elasticity doubles the short-run elasticity. However, it presents no evidence to support this assumption.

4.4 Speed of adjustment

An issue related to the difference in size between the elasticity in the short run and that in the long run is the speed with which the short-run elasticity reaches the long-run elasticity. This has important implications because the revenue reimbursement due to operators decreases as the speed of adjustment increases.

The DfT's recommended profile for adjustment is shown in Table 4.2. As in the case of other assumptions in the DfT guidance, limited evidence is provided to support the proposed speed of adjustment.

Table 4.2 DfT-recommended rate of change over time

One year	Two years	Three years	Four years	Five years
50%	75%	88%	94%	100%

Source: DfT (2007), op. cit.

One factor that might affect the speed of adjustment is the existence of asymmetries in passenger responses to fare increases/decreases. This may be because passengers are quicker to adjust to fare increases (users will notice fare increases almost instantly) than to fare decreases (non-users may take longer to be informed of fare decreases; also, non-users may have become captive to another mode, such as car travel). Dargay and Hanly (1999) present some evidence to suggest more rapid response to fare increases than to fare decreases for county-level data. An Oxera study, albeit for rail travel, provided some evidence on the asymmetric effects of passengers responding to changes. This study found that, in four of the six cases examined, there was a greater short-run response to price rises than to falls.¹⁶

If the true speed of adjustment were slower than recommended by the guidance or the difference between short- and long-run elasticity were not significant, this would mean that higher long-run elasticities in later periods would not be implemented (so soon), and therefore that the revenue reimbursement would be higher (for a longer period).

¹⁴ Goodwin, P. (2006), 'Concessionary Travel Schemes Appeals: Further Comments', report to Stagecoach plc, September.

¹⁵ This is, however, not empirically analysed in Goodwin (2006).

¹⁶ Oxera (2005), 'How do Rail Passengers Respond to Change', prepared for Passenger Demand Forecasting Council.

5 Other assumptions that may affect the revenue reimbursement

In addition to the assumptions regarding the shape and the elasticity of the bus travel demand, other aspects of the RAT could affect the revenue reimbursement perceived by bus operators.

5.1 Trip rate of existing and new pass-holders

The DfT suggests that the trip rate of new pass-holders is lower than that of existing pass-holders. This is incorporated in the RAT by allowing a comparison between the trip rate of existing pass-holders and that of new pass-holders. In support of this assumption, the DfT quotes the National Travel Survey, which shows that current non-pass-holders in England (excluding London) undertake around 14% of the trips, compared with those with a concessionary pass. While this evidence suggests that non-pass-holders travel less than pass-holders, it does not necessarily indicate that, when non-pass-holders become new pass-holders, their trip rate will still be 14% of the trips made by existing pass-holders.

5.2 Changes in demand drivers over time

One of the implications of the assumption of a negative exponential demand function is that information on changes in demand drivers is not required to update the elasticity over time. In the RAT, patronage trends, quality changes and changes in substitutes are important for the calibration of the elasticity in the year before the scheme. After that, the only parameters required to update the elasticity are the change in real fare and the assumptions regarding the speed of adjustment of the elasticity in the long run.

In practice, however, changes in demand drivers would certainly have an impact on the fare elasticity over time. It is therefore recommendable to account for changes in demand drivers when updating the elasticity in future periods.

5.3 Inflation rates

In the RAT, the elasticity is calculated with respect to changes in real fares. Real fares are calculated in the RAT by deflating fares in later years based on an estimate of inflation. The inflation assumption for further years is hard-coded in the RAT at 2.5%. It is not possible to change this assumption in the RAT, although it might be recommendable to update this figure in line with changes in government forecasts.

6 Conclusions

The DfT guidance provides a framework for reimbursement bus operators for the provision of free concessionary travel. The core principle of 'neither better nor worse off' is accomplished by compensating the operators for the revenue forgone and the additional cost imposed by the generated trips. In determining this forgone revenue and the additional cost, the DfT uses a number of assumptions that could affect the reimbursement due to operators. A general observation of the assumptions used by the DfT is that they are not justified by robust evidence.

Based on a review of the literature and evidence on bus transport demand, and on the numerical example of the impact in reimbursement due to changes in the DfT assumptions, the following conclusions emerge.

- The DfT methodology is based on assumptions about the nature of demand for bus travel. These assumptions are fairly arbitrary, cannot be justified unambiguously by empirical evidence, and involve extrapolating experience with current levels of prices to zero prices in a way that is highly dependent on the precise method used.
- While there is some evidence on the use of a demand function with a variable elasticity of demand (eg, a negative exponential function), other functional forms with variable elasticity of demand could also be used (eg, a linear demand). If other functional forms are used, the DfT's proposed revenue reimbursement formula does not hold for estimating the number of non-generated trips.
- The evidence presented by the DfT to support its assumption that the impact of a 1% increase in real fares is 0.2% is limited. This assumption has important implications for the bus operators, as values lower than the assumed 0.2% could have a large impact on the reimbursement factor. There is also a potential inconsistency in the application of this adjustment as it could mean that the DfT is using two different demand curves: one for calculating the elasticity and one for calculating the effects of the scheme.
- The evidence presented by the DfT to assume that the long-run elasticity doubles the short run elasticity is limited. Related to this point, little evidence is presented to justify that the effect of the scheme follows a specific speed of adjustment over a five-year period.
- On the cost side, no support is provided to sustain the assumption that the range of additional costs is between £0.01 and £0.15 per generated trip.

A1 Elasticity calculation

This appendix shows that the term:

$$-\frac{\ln\left(\frac{q_3}{q_2}\right)}{f_2}$$

in the elasticity at full fare (Equation 4.2) is equivalent to the parameter $-b$ in the demand equation $q = e^{-b \cdot f}$

$$\begin{aligned} -\frac{\ln\left(\frac{q_3}{q_2}\right)}{f_2} &= -\left[\frac{\ln(q_3) - \ln(q_2)}{f_2}\right] \text{ (by law of logarithms)} \\ &= -\left[\frac{-b \cdot f_3 - (-b \cdot f_2)}{f_2}\right] \end{aligned}$$

(Using the fact that demand:

$$q = e^{-b \cdot f}$$

could be rewritten as:

$$\ln(q) = -b \cdot f$$

$= -b$ (because f_3 is the zero fare and f_2 cancels out).

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